Finite-frequency sensitivity kernels of seismic waves to fault zone structures

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SUMMARY
We analyse the volumetric sensitivity of fault zone seismic head and trapped waves by constructing finite-frequency sensitivity (Fréchet) kernels for these phases using a suite of idealized and tomographically derived velocity models of fault zones. We first validate numerical calculations by waveform comparisons with analytical results for two simple fault zone models: a vertical bimaterial interface separating two solids of differing elastic properties, and a ‘vertical sandwich’ with a vertical low velocity zone surrounded on both sides by higher velocity media. Establishing numerical accuracy up to 12 Hz, we compute sensitivity kernels for various phases that arise in these and more realistic models. In contrast to direct P body waves, which have little or no sensitivity to the internal fault zone structure, the sensitivity kernels for head waves have sharp peaks with high values near the fault in the faster medium. Surface wave kernels show the broadest spatial distribution of sensitivity, while trapped wave kernels are extremely narrow with sensitivity focused entirely inside the low-velocity fault zone layer. Trapped waves are shown to exhibit sensitivity patterns similar to Love waves, with decreasing width as a function of frequency and multiple Fresnel zones of alternating polarity. In models that include smoothing of the boundaries of the low velocity zone, there is little effect on the trapped wave kernels, which are focused in the central core of the low velocity zone. When the source is located outside a shallow fault zone layer, trapped waves propagate through the surrounding medium with body wave sensitivity before becoming confined. The results provide building blocks for full waveform tomography of fault zone regions combining high-frequency head, trapped, body, and surface waves. Such an imaging approach can constrain fault zone structure across a larger range of scales than has previously been possible.

Key words: Interface waves; Guided waves; Seismic tomography; Wave propagation; Dynamics and mechanics of faulting; Fractures and faults.

1 INTRODUCTION
The subsurface structure of fault zones remains a critical target for seismic imaging. Accurate derivation of fault zone properties can provide fundamental information for numerous applications ranging from earthquake properties and fault mechanics to geophysical exploration. In particular, bimaterial interfaces separating faster and slower media can strongly affect properties of earthquake ruptures and seismic ground motion (e.g. Ben-Zion 2001; Ampuero & Ben-Zion 2008; Brietzke et al. 2009); damage zone characteristics can be used to infer background and dynamic stress fields during rupture propagation, effective brittle rheology and other aspects of faulting (e.g. Wilson et al. 2003; Ben-Zion & Shi 2005; Dot et al. 2006; Templeton & Rice 2008; Faulkner et al. 2010; Xu et al. 2012, 2015). Seismic deployments near fault zones have allowed seismologists to confirm the existence of fault zone head and trapped waves (e.g. Li & Leary 1990; Ben-Zion & Malin 1991) predicted from theory for bimaterial interfaces and coherent low velocity fault zone layers (e.g. Ben-Zion 1989; Ben-Zion & Aki 1990). In addition to producing specific phases, these structural elements perturb the general features of body and surface waves near faults (e.g. Graves et al. 2011).

Fault zone head waves propagate along bimaterial interfaces and are analogous to the Pn crustal phase that refracts along the Moho boundary. For earthquakes near the interface, first-arriving head waves at stations in the slower medium have opposite first motion polarity from the following more impulsive direct P arrivals (e.g. Ben-Zion 1989, 1990). When unrecognized, these characteristics can distort and bias solutions for focal mechanisms and earthquake locations derived from near-fault data (e.g. McNally & McEvilly 1977; McGuire & Ben-Zion 2005). On the other hand, proper use of head waves can increase the resolution of local tomographic images.
and earthquake properties (e.g. Ben-Zion et al. 1992; Zhao et al. 2010; Bennington et al. 2013).

Trapped waves arise from constructive interference within a low-velocity fault zone layer leading to resonances modes analogous in their basic form to traditional Love waves (e.g. Ben-Zion & Aki 1990; Li & Leary 1990). Similar to wave propagation effects in narrow basins, trapped waves may amplify ground motion by a factor of 5 or more near the fault (e.g. Cormier & Spudich 1984; Rovelli et al. 2002; Avalone et al. 2014). Modelling trapped waves can provide high-resolution information on the spatial extent, continuity and velocity reduction of core fault damage zones (e.g. Li et al. 1990, 1994; Peng et al. 2003; Lewis & Ben-Zion 2010). Analytical and numerical parameter-space studies of wave propagation in fault zone models in 2-D (e.g. Ben-Zion 1998; Yang & Zhu 2010) and 3-D (e.g. Igel et al. 1997, 2002; Jahne et al. 2002) have indicated strong trade-offs among various model parameters (e.g. degree of velocity contrast, fault zone width, propagation distance within the low velocity zone, sharpness of the boundaries, attenuation) leading to non-uniqueness in data interpretation.

Although fault zone waves are limited to relatively narrow regions, they have been observed at an increasing number of near-fault deployments. Recent examples include the Calico (Cochran et al. 2009; Hillers et al. 2014), San Jacinto (Yang et al. 2014; Ben-Zion et al. 2015) and Hayward faults (Allam et al. 2014a) in California, the rupture zone of the 2009 L’Aquila earthquake in Italy (Calderoni et al. 2010), the Mudurnu segment of the North Anatolian fault zone (Bulut et al. 2012) and the Garzé–Yushu fault in the Tibetan Plateau (Yang et al. 2015). See Lewis & Ben-Zion (2010) for earlier examples. Modelling these waves in a broad regional context requires advanced 3-D numerical simulations and efficient inversion techniques. It is well-recognized that there are broad connections (Tromp et al. 2005) between related topics of finite-frequency kernels (Marquering et al. 1999), time-reversal imaging (Fink 1992), and adjoint methods (Tarantola 1984; Talagrand & Courtier 1987). The key feature is that the accuracy of 3-D seismic wavefield simulations can be used directly within an inverse problem, allowing for ‘wiggles’ on seismograms to be translated into unknown structural variations with minimal assumptions. These principles were demonstrated in Tape et al. (2007) and applied at a large computational scale for southern California by Tape et al. (2009, 2010) and Lee et al. (2014). The approach of iteratively improving a 3-D reference model using wavefield simulations is commonly referred to as adjoint tomography, full waveform tomography, or full waveform inversion.

Full waveform tomography incorporates sensitivity kernels (e.g. Dahlen et al. 2000) computed from accurate 3-D numerical simulations of the seismic wavefield as the forward portion of the inverse problem, relating changes in material properties to specific changes in observed seismic data. Accounting for frequency-dependent effects of the full wavefield has been shown to improve imaging resolution (e.g. Montelli et al. 2004; Sigloch et al. 2008). These methods have been successful at both regional (e.g. Tape et al. 2009, 2010), and continental scales (Fichtner et al. 2009; Zhu et al. 2012; Fichtner et al. 2013), and have been proposed at the global scale (Boschi et al. 2007). Advances in tomographic techniques allowed for the extraction of information from low-amplitude portions of the wavefield (e.g. Gung & Romanowicz 2004), measurement of anisotropy (Zhu et al. 2013) and attenuation (Fichtner et al. 2010), and creation of more stable misfit functions (Bozdag et al. 2011); all of these are important aspects of imaging fault zone structure (e.g. Blakelock et al. 1989; Ben-Zion 1998; Lee et al. 2014).

Attenuation can significantly affect the properties of seismic waveforms in general and fault zone waves in particular (e.g. Ben-Zion 1998) by limiting the bandwidth of recorded data. This limits the imaging potential of observed seismic waves. Attenuation within adjoint-based inversions has long been identified as possible, yet challenging, both in terms of implementation and computational costs (Tarantola 1988; Tromp et al. 2005). Recent efforts using 3-D models have demonstrated the feasibility of inverting for attenuation structure (Fichtner & van Driel 2014; Zha et al. 2015). Our emphasis for fault-zone imaging is to generate synthetic fault zone waveforms that resemble observed ones; future efforts to match these waveforms accurately will likely require the implementation of attenuation in the forward and inverse modelling.

In this work, we analyse the 3-D sensitivity of fault zone head and trapped waves to fault zone structure and discuss the effects of structural variations on the generated wavefield. This can provide a basis for incorporating the sensitivity kernels of fault zone phases into high-resolution full waveform inversions of local fault zone structure. Our basic goal is to develop a framework for a full fault-scale tomographic inversion incorporating body waves, surface waves, and fault zone phases. Towards this goal, we first validate the 3-D spectral element code of Komatitsch & Tromp (1999) for the fault zone scale by comparing basic model simulations with analytical results (Ben-Zion 1990; Ben-Zion & Aki 1990), and then examine the finite-frequency sensitivity of fault zone seismic head and trapped waveforms to specific properties of fault zone structure, including low velocity zone width, degree of contrast, boundary sharpness, and employed frequency. The study follows previous efforts to reveal the sensitivities of seismic waveforms to the structural medium (e.g. Liu & Tromp 2006, 2008; Sieminski et al. 2007a,b); such efforts are aimed at guiding measurement selection and model parameterization for tomographic inversions. The fault zone sensitivity kernels provide a foundation for future inversion of seismic data in southern California that builds upon the efforts of Tape et al. (2009) by including a much larger dataset of high-frequency near-fault waveforms, and Allam & Ben-Zion (2012) by providing a more accurate representation of the wavefield sensitivity (kernels instead of rays) and incorporating fault zone waves.

2 METHODS

The general workflow for the computation of sensitivity kernels is as follows:

1. Meshes for the various fault zone models are constructed using GEOCUBIT (Casanotti et al. 2008), a spectral element mesher optimized for high-performance computing clusters and the specific difficulties of earth models (e.g. large variations in seismic velocity and geometrical complexity).

2. The full elastic wavefield is computed using SPECFEM3D (Komatitsch & Tromp 1999), using a point-source in space and time (or a series of simultaneous point sources to approximate a line source).

3. Synthetic seismograms are extracted at station locations, measured and filtered to numerical accuracy (as described in Section 2.3.3), and windowed around selected phases of interest.

4. P and S velocity sensitivity kernels are computed using adjoint methods (Tromp et al. 2005; described in Section 2.1) and plotted using open-source 3-D visualization software paraview (www.paraview.org).
Table 1. Convention for coordinate axes. The slow layer in the vertical sandwich model is between $x = 0$ km and $x = 1$ km (or $x = 0.5$ km).

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Description</th>
<th>Zero value</th>
</tr>
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<tbody>
<tr>
<td>$x$</td>
<td>Fault-normal direction, $\hat{x}$ points toward slower block</td>
<td>$x = 0$ is the fault interface (of faster block)</td>
</tr>
<tr>
<td>$y$</td>
<td>Fault-parallel direction, $\hat{y}$ points toward $-(\hat{x} \times \hat{z})$</td>
<td>$y = 0$ is the boundary of 3-D model for simulations</td>
</tr>
<tr>
<td>$z$</td>
<td>Vertical direction, $\hat{z}$ points up</td>
<td>$z = 0$ is the surface</td>
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2.1 Sensitivity kernels

Here we briefly review some key formulations for the computation of sensitivity kernels. For more detailed discussions about kernel computation, see Tromp et al. (2005), Liu & Tromp (2006) and Tape (2009).

Sensitivity kernels provide a measure of the relationship between a given model parameter (e.g. P-wave velocity) and a given wave-form property (e.g. traveltime) for a single event recorded at receiver $r$. We follow the methods of Tromp et al. (2005) to construct sensitivity kernels, which are based on the interaction of a forward wavefield $s(x, t)$ and a time-reversed adjoint wavefield $s^*(x, x_r, T - t)$. For example, the sensitivity kernel for the bulk modulus $K$ is given by

$$K_{s^*}(x, x_r) = -\int_0^T \kappa(x) [\nabla \cdot s(x, x_r, T - t)] [\nabla \cdot s(x, t)] \, dt,$$

where $x_r$ is the receiver location. Similar expressions can be derived for the shear modulus $\mu$ and density $\rho$, which leads to the expression of sensitivity kernels for the velocities of compressional ($V_p$) and shear ($V_s$) waves,

$$\kappa_{V_p} = \left(\frac{x + \frac{2}{3} \mu}{\kappa}\right) K_p, \quad \kappa_{V_s} = 2 \left(\kappa_{\mu} - \frac{4}{3} \kappa K_s\right),$$

with which we are concerned for the present work. Also of interest is the waveform adjoint source,

$$f^*(x, t) = w_r(T - t) \partial_t s(x, x_r, T - t) \delta(x - x_r),$$

where $w_r(t)$ is a time window chosen for a seismogram at receiver $r$. The adjoint wavefield is created from the forward and time reversed wavefield at the receiver location. We compute both $s(x, t)$ and $s^*(x, x_r, T - t)$ numerically, assigning point sources at source and receiver locations, and using step functions in source–time. By applying a filter to either $s$ or $s^*$, a frequency-dependent kernel can be computed, expressing structural sensitivity to a specific waveform frequency. In general, we apply a low-pass filter with corner frequency set at the numerical resolution and thus calculate broadband sensitivity kernels, though frequency-dependent sensitivity is examined in Section 3.3.

One advantage of this method is that no a priori knowledge of seismic phase is necessary in order to construct a kernel. One can simply pick any arbitrary $w_r(t)$ on the forward seismogram and then calculate $K_p$ and $K_{\mu}$. The kernels themselves provide insight into the phases present within the selected time window. This is ideal for the case of fault zones, where even simple idealized structures can create complicated wavefields. In this study, we use this property to ascertain which portion of the model structure is being illuminated by the selected seismogram windows.

2.2 Mesh construction and numerical simulations

Meshing is a non-trivial part of the workflow when using spectral-element methods, and must be guided by two key objectives. First, we must accurately represent the structural model by honouring known interfaces (e.g. bimaterial) and by satisfying numerical stability conditions in the frequency range of the target waveforms. Second, the designs must be computationally efficient. Because resolvable frequency of each element depends on the seismic velocity of the element, employing larger elements for portions of the model with faster wave speeds can lead to immense reductions in computation time.

We generate hexahedral meshes using GEOCUBIT (Casarotti et al. 2008; Peter et al. 2011), a python-based extension of the meshing package CUBIT (www.cubit.sandia.gov) for geological structures. We modify GEOCUBIT to allow for sharp vertical and near-vertical interfaces and to allow mesh densification in the low velocity portions of the fault zone model. We employ a right-handed fault-centric coordinate system; $x$ is the fault-normal, $y$ the fault-parallel coordinate, and $z$ is depth negative (Table 1). Though our simulations employ a right-handed system, in the figures and text of Section 3 we both plot and refer to ‘depth’ in the traditional sense of positive downward with a value of 0 at the surface.

The three meshes we employ are shown in Fig. 1(a): 25 km $\times$ 20 km $\times$ 20 km bimaterial mesh, a dense 25 km $\times$ 20 km $\times$ 10 km vertical sandwich for the examination of high-frequency waves (Section 3.1), and a 20 km $\times$ 20 km $\times$ 40 km mesh for use with several more complicated models of fault zone structure (Sections 3.2–3.4). Numerical resolution is measured for the first two cases by comparison to analytical solutions.

2.3 Comparison to analytical solutions

It is an important starting point in studies of numerical wave propagation to benchmark results against known analytical solutions. For fault zone phases, we use two such relevant analytical solutions: a bimaterial interface separating two half spaces (Ben-Zion 1990), and a ‘vertical sandwich’ low-velocity layer separating two higher velocity quarter spaces (Ben-Zion & Aki 1990). The details of the station configuration, free surface and source parameters are slightly different for the two solutions. We explicitly compute numerical accuracy at each synthetic receiver location by calculating the correlation coefficient between analytical and numerical waveforms as a function of frequency. The waveforms, sensitivity kernels, and numerical accuracy resulting from these analytical solutions provide basic intuition for the interpretation of more realistic models examined in Section 3, which are essentially incrementally more complex versions.

2.3.1 Bimaterial interface

Ben-Zion (1990) presented a 3-D solution for the displacement response of two welded half-spaces of differing elastic properties to a general point dislocation occurring on the bimaterial interface. This solution and the earlier 2-D version (Ben-Zion 1989) predicted the existence of fault zone head waves, which have been repeatedly observed in nature (e.g. Zhao et al. 2010; Allam et al. 2014a, and references therein). Using the mesh shown in Fig. 1(a), we place a
Sensitivity kernels of fault zone waves

Figure 1. Hexahedral meshes for seismic wave propagation in idealized fault models. (a) $25 \times 25 \times 20$ km bimaterial model of two welded quarter spaces, with 200 m constant grid spacing. (b) ‘Vertical sandwich’ model of a fault zone characterized by a slow velocity layer between two quarter spaces. Mesh dimensions are $20 \times 25 \times 10$ km, with a fault zone that is 1 km wide. The grid spacing is 250 m outside and 83 m inside the low velocity zone. (c) Mesh used for miscellaneous models, shown here with the tomography model parameterization. Grid spacing is 500 m outside and 83 m inside the low velocity zone. (d) Inset showing the densification of the mesh inside the low velocity zone. The lines shown in a, b, and c mark the locations of the stations shown in later figures.

Figure 2 shows displacement record sections for the bimaterial case of the vertical component of motion for the numerical simulation (red) compared to the analytical solution (black). All traces are low-pass filtered with a cut-off frequency of 7 Hz and normalized by a constant value (i.e. the relative amplitudes are directly comparable). The calculated arrival for the direct P and S waves (vertical black lines) are also shown for reference. In general, the fits between numerical and analytical seismograms are good for all phases and all components with correlation coefficient $R \geq 0.98$ for all traces.

As expected for the prescribed geometry, stations on the faster side generally feature only simple P and S phase arrivals with the correct polarity. For stations near the fault on the slow side, the first arrival is an emergent phase with the polarity of the opposite side P wave, followed by a normally polarized P wave. This emergent phase is the head wave which propagates with the velocity of the faster medium and arrives before the direct P wave. Farther from the fault on the slow side, the head wave is absent and the first arrival is the direct wave. A clear S head wave is also observed on the slow side of the fault, though this phase has never been identified in real seismic data since it tends to be obscured by various other phases (e.g. P-to-S conversions, late P arrivals, etc.).

Fig. 4 shows sensitivity kernels for several select phases observed for the bimaterial case, for the windows defined in Fig. 3. The direct P wave on both sides of the fault (Figs 4a and b) far away from the interface produces classical banana-doughnut kernels; early arrivals are unaffected by the interface and propagate as in a homogeneous solid. For a near-fault station on the slow side, the P head wave has an asymmetric ‘canoe’ shaped kernel with sensitivity largely on the fast side (Fig. 4c). The S head wave has sensitivity as the P head wave, but with additional late-arriving reflections (Fig. 4d). The subsequent direct S at the same station has sensitivity predominantly on the slow side. Altogether, the various phases illuminate the entire bimaterial structure.

2.3.2 Vertical sandwich

Ben-Zion & Aki (1990) presented a solution for the displacement response to an infinite line dislocation in the $y$-direction along the edge of a vertical low velocity zone separating materials of differing elastic properties (‘vertical sandwich’). The source–receiver geometry and material properties of the numerical model used for this configuration (without a velocity contrast on the opposite sides of
the fault zone) are shown in Fig. 2(b). In this model configuration, the assumed line source generates only motion in the fault-parallel (y) direction. To mimic the infinite extent of the line source, we employ a mesh elongated in the y direction (Fig. 1b) and place a series of 1000 right-lateral point sources at regular 40 m spacing along the left interface of the low velocity zone at a depth of 14 km. The velocity in the 500-m-wide inner layer is reduced by 40 per cent compared to the fastest medium 1; this is a high but reasonable value compared to previous modelling of observed trapped waves (e.g. Lewis & Ben-Zion 2010). By using a mesh refined only inside the low velocity zone, we gain numerical accuracy and reduce computation cost. Receivers are placed inside the mesh volume in a fault-normal array above the line source.

Fig. 5 shows y-component velocity record sections for the vertical sandwich case. All traces are shown on the same amplitude scale and are low-pass filtered with a cut-off frequency of 6 Hz, a value made possible in the low-velocity zone by the refined meshing. At this frequency, the correlation coefficient between numerical and analytical seismograms $R$ is greater than 0.98 for all traces. Outside of the low velocity zone (blue dashed lines), the seismograms are simple with only direct S arrivals. Inside the low velocity zone, more complex reflected phases and trapped modes develop; these are
Figure 4. Gallery of sensitivity kernels for the bimaterial model (Figs 1a and 2a). The receivers are located at depth, but the surface projections are shown for reference (purple boxes). (a) $P$ wave kernel on the faster side showing traditional banana-doughnut shape. (b) $P$ wave kernel on the slower side; the direct $P$ wave kernel is unaffected by the fault structure. (c) $P$ head wave kernel with sensitivity concentrated on the fast side. (d) $S$ head wave kernel, which also contains added reflections and phase conversions. The $S$ head wave kernel has narrower width than the $P$ head wave kernel, as expected from the direct relationship between kernel width and seismic velocity.
Figure 5. Y-component velocity seismograms for the vertical sandwich model (Figs 1b and 2b), low-pass filtered with 6 Hz corner frequency. The line source is in the y-direction at \( x = 0 \) km, \( z = -14 \) km. Black seismograms are based on the analytical solution of Ben-Zion & Aki (1990), and red seismograms are the numerical results. Because of the geometry of the structure and line source, only \( S \) body waves and fault zone trapped waves are present.

Figure 6. Comparisons of numerical-analytical correlation coefficient versus low-pass corner frequency for (a) the bimaterial and (b) the vertical sandwich models. Each blue curve represents a seismogram recorded at a single station. Red dashed lines show the frequency axis intercept where each curve’s correlation coefficient equals 0.98; frequencies below this value are considered resolved numerically. The resulting distributions of resolved frequency are shown (light blue rectangles). The scatter in numerical accuracy is due to differences in velocity structure and spatially variable mesh grid size.

2.3.3 Quantifying numerical accuracy

In order to quantify numerical accuracy, we compute the correlation coefficient between the analytical and numerical solutions at each station as a function of frequency. We use this approach because the numerical accuracy is not a constant value for a single mesh; it depends on the velocity, mesh size and incoming wave frequency and thus varies spatially within a single velocity model. The correlation coefficient is

\[
R = \frac{\text{cov}(v_a, v_n)}{\text{std}(v_a) \text{std}(v_n)} = \frac{v_a \cdot v_n}{\|v_a\| \|v_n\|},
\]

where \( v_a \) is the seismogram from the analytical solution and \( v_n \) is the seismogram from the numerical result.

Fig. 6 shows \( R \) as a function of low-pass corner frequency for the (a) bimaterial and (b) vertical sandwich cases. For both cases, the correlation is very close to 1.0 at low frequencies and begins to decay above a certain corner frequency. For the bimaterial case, the less well-resolved stations are closest to the boundaries of the simulation. The vertical sandwich case has a larger spread in numerical accuracy because of the variable mesh spacing; the two stations which stand out with the highest numerical resolution are both located inside the low velocity zone. Though this region has the lowest velocity, it has the densest mesh, making it the best-resolved region of the model.

Fig. 7 shows a comparison of the solutions for the vertical sandwich case at one of these stations as a function of frequency. The trapped waves display a dispersive pattern similar to surface waves, with higher frequencies arriving later. Numerical noise begins to become prominent above 15 Hz, and is visible as high-frequency oscillations after the main phase arrivals. The character of the
We choose a threshold of 0.98 for estimating the numerical accuracy. Perfectly, while at higher frequencies, noise dominates the numerical solution. Corner frequency. At low frequencies, the two traces correlate almost perfectly, while at higher frequencies, noise dominates the numerical solution. We choose a threshold of 0.98 for estimating the numerical accuracy.

Numerical noise shown by this comparison is a useful tool to estimate numerical accuracy for the models discussed in Section 3 for which no analytical solution exists. Based on the correlations observed for this and other stations, we choose a value of 0.98 as the threshold for determining the low-pass corner frequency; we consider seismograms at or above this value to be well-resolved numerically. Based on this criterion, the distributions of well-resolved frequencies shown in Fig. 6 illustrate the large variation in numerical accuracy that can be observed for a single mesh.

3 ADDITIONAL RESULTS

In this section, we investigate the 3-D sensitivity of seismic waves to particular aspects of fault zone structure by computing sensitivity kernels in different velocity models. First, we show results from a vertical sandwich model with three different media, a left-lateral point source, and a free surface. This model produces all four of the major phases observed inside the low velocity zone (Fig. 9b); these values are consistent with observations of 40 per cent velocity reduction in fault zones (e.g. Ben-Zion et al. 2003; Lewis & Ben-Zion 2010), and a 15 per cent across-fault velocity contrast (e.g. McGuire & Ben-Zion 2005; Allam et al. 2014b). The mesh and model allow numerical accuracy of approximately 10 Hz in the central portion of the model, based on the numerical-analytical comparisons described in Section 2.3.3. The left-lateral point source is placed on the left interface of the low velocity zone at a depth of 5 km. An array of 70 stations is placed on the free surface from −7 km to 10.25 km in the x direction, with a spacing of 250 m. The along-fault (y) distance between sources and receivers is 11.5 km.

Fig. 8 shows vertical-component displacement seismograms low-pass filtered with a corner frequency of 10 Hz. Although the vertical sandwich is a simple 1-D model, it gives rise to a complex seismic waveform with multiple reflected and refracted phases including body waves, surface waves, head waves and trapped waves. Stations on the faster (left) side of the fault generally have cleaner, more impulsive arrivals than stations on the slower side, though near-fault stations on both sides of the fault feature considerable complexity. Reflected and converted phases are visible after the S wave on both sides of the fault. Late-arriving trapped waves are by far the largest amplitude phases observed inside the low velocity zone, and they appear to persist for some distance outside the zone on both sides. However, the trapped waves visible outside of the low velocity zone are lower in amplitude by an order of magnitude than the trapped waves inside the fault zone; their appearance here is due to the normalization used for plotting. Head waves on the slow side of the fault and within the low velocity zone are emergent arrivals with much lower amplitude and opposite polarity to the direct body waves. Nevertheless, the head wave signals are clear when the traces are amplified (inset). Spurious reflections from the imperfect absorbing boundaries are visible as late arrivals with a negative moveout; because they are low in amplitude and arrive after the phases of interest, they are ignored in further analysis.

Though the waveforms are complicated, several of the observed phases have simple sensitivity kernels. Fig. 9 compares the sensitivity kernels for the phases highlighted in Fig. 8. The direct P wave (Fig. 9a) on the fast side of the fault is a traditional banana–doughnut kernel for a homogeneous medium, similar to that of the bimaterial case. The canoe-shaped P head wave observed in the low velocity zone (Fig. 9b) is similar to that of the direct wave, but has asymmetric sensitivity amplified on the faster side of the fault. The trapped wave kernel (Fig. 9c) has narrowly focused sensitivity constrained to within the low velocity zone itself. Because of its flattened appearance, we term this a ‘pancake’ kernel. Unlike the head wave kernels, the trapped wave kernels have virtually no sensitivity outside the fault zone.

Fig. 9(d) shows the sensitivity of surface waves to the vertical sandwich model. The fast-side surface wave kernel (left solid) features broad but shallow sensitivity outside of the fault zone. Inside the fault zone, the surface wave kernel appears to contain multiple reverberations between the source and the surface. The combined
effects form a striking kernel that could be described as two pancakes: one flat-lying at the surface that abruptly terminates at the fault zone, and one confined to the fault zone. The kernel is comparably sensitive to small-scale variations that are shallow and outside the fault zone and to small-scale variations inside the fault zone.

3.2 Frequency dependence

To determine the frequency-dependent sensitivity of trapped waves, we compute sensitivity kernels in eight different frequency bands for the vertical sandwich model. The mesh, source–receiver geometry, and velocity model used are as described in Section 2.3.2 (shown in Figs 1b, d and 2b) with a few changes: the top surface of the model is a free surface, the source is a left-lateral point source located at $y = 10$ km and $z = 9$ km, and the receivers are placed on the free surface. It should be noted that the source is on the left boundary rather than centred in the low velocity zone, breaking the symmetry. We select eight frequency bands from 1 to 8 Hz, applying a 2-pass Butterworth filter to the adjoint source described in eq. (3). We select the last-arriving highest-amplitude phase from the seismogram nearest to the source inside the low velocity zone. The full seismograms, examined frequencies, and phases picked are shown in Fig. 10(a).

Fig. 10(b) presents cross-sections in the source–receiver plane of the frequency-dependent trapped wave sensitivity kernels. The kernels show the same pattern as those of Love waves (e.g. Zhou et al. 2004), with high negative sensitivity along the direct propagation path and surrounding lobes of alternating polarity corresponding to the Fresnel zones (e.g. Knapp 1991). With increasing frequency,
Figure 9. Sensitivity kernels computed from the phases highlighted in Fig. 8. The kernels in (a) and (c) are for a station outside the fault zone at $x = -6.25$ km; the kernels in (b) and (d) are for a station at inside the fault zone at $x = 0.25$ km. (a) Fast side $P$ wave sensitivity kernel with classical banana-doughnut structure. (b) $P$ head wave kernel with sensitivity mostly on the fast side. (d) Surface wave kernel has shallow sensitivity except inside the low velocity zone. (e) Trapped wave kernel is sensitive entirely to the low velocity zone and is similar in appearance to the surface wave kernel.

The overall width of the kernel decreases and the number of lobes increases. Fig. 10(c) shows fault-normal cross-sections through the kernels at $y = 7.5$ km, $z = 5$ km. The sensitivity kernels also narrow in the fault-normal direction, but much more quickly as frequency increases; higher frequency waves become more localized in the low velocity layer and more sensitive to it, up to a maximum frequency controlled by the thickness and velocity of the layer and source frequency content. Thus, the sensitivity of fault zone trapped waves...
is analogous to that of Love waves arising in the context of global seismology, which have rapidly diminishing frequency-dependent sensitivity below the Moho.

3.3 Fault zone boundary smoothing

Here we examine models in which the boundaries of the vertical sandwich are smoothed to varying degrees to determine the effect on sensitivity kernels. The mesh, source–receiver geometry, and general velocity model used are as described in Section 3.2. The base velocity model is the same as that shown in Fig. 1(b), but smoothed in one of two ways. In the first series of models, we apply Gaussian smoothing with incrementally increasing half-width (Fig. 11a). In this method, the integral of velocity reduction over the fault zone is conserved, while the maximum amplitude of the reduction decreases. In the second series of models, we hold the value of the lowest velocity fixed and we apply smoothing only to the boundaries (Fig. 11b). The result is that the core region of low velocity remains unchanged, but with widening edges. These two types of velocity models reflect conflicting viewpoints about the macroscale structure of fault damage zones (Faulkner et al. 2010).

Fig. 11 shows fault-normal profiles along the smoothed velocity models (a, b), the shear velocity sensitivity along the same profiles (c, d) and the windows of the seismograms used to compute them (e, f). The seismograms are vertical component velocity waveforms, low-passed at 6 Hz for the station nearest to the left-most boundary inside the low-velocity zone. The corner frequency of the low-pass filter is set relatively low in order to ensure comparable numerical accuracy; in general, models with higher minimum shear wave speeds will be accurate to higher frequencies. The shear wave sensitivity kernels were computed in all cases for the last clear arrival
after the body waves. The smoothing parameter shown in the key is the half-width ($\sigma$) of the Gaussian kernel in numbers of gridpoints (80 m each).

The waveforms from the pure Gaussian smoothing models (Fig. 11c) show strong variations as the smoothing increases. The amplitude, arrival time, and number of oscillations of the trapped waves decrease with increasing smoothing. Although the total amount of low velocity material is conserved between the different models, the smoother models are less effective at exciting trapped waves. Even the change from no smoothing to $\sigma$ is dramatic, resulting in greatly decreased amplitude. In contrast, the relative amplitudes of the head waves and body waves are similar regardless of the model. As shown previously, these phases have very little sensitivity to the parameters of the low velocity zone, instead having sensitivity almost exclusively to the faster medium (Fig. 9b).

Important differences become clear by examining fault-normal 1-D profiles through the kernels (Fig. 11c). In general, the kernels become broader as the smoothing increases, reflecting the increasing low velocity zone width. At low smoothing, kernels have very simple profiles with no sensitivity outside of the fault zone and negative sensitivity (which means that an increase in velocity would decrease travel time) within it. However, kernels from the smoothest models have more complicated oscillatory structure. At the highest smoothing, a lobe of positive sensitivity on the left edge of the low velocity zone indicates that an increase in velocity would cause the traveltime of the entire trapped wave package to increase.

The effects of boundary smoothing for a constant low-velocity layer (Fig. 11b) are less pronounced. The waveforms (Fig. 11f) show the same trend towards lower amplitude and fewer trapped wave oscillations with increasing smoothing, but the effect is more subtle. Even though the total velocity is decreasing with increasing smoothness, trapped waves are still efficiently excited by a low velocity zone with sharp boundaries. Head waves and direct body waves are only slightly affected by boundary smoothing. The profiles of the sensitivity kernels (Fig. 11d) show some surprising features. The width of the sensitivity remains almost unchanged as smoothing increases. This indicates that even though the fault zone width is increasing, the highly reduced core is the part responsible for the character of the trapped waves; the trapped waves are primarily sensitive to the lowest-velocity portion of the fault zone.

We also see that these kernels become oscillatory at higher smoothing, similar to the case of pure Gaussian smoothing models. Even though the trapped waves appear similar in waveform between the $\sigma$, $7\sigma$ and $9\sigma$ models, the sensitivity kernels are very different. Because trapped waves are the result of the interference between multiple internally reflected phases, even simple changes in model parameters can create substantial differences in sensitivity.

### 3.4 Realistic models

We now consider some more complicated and realistic 3-D velocity models. Using the same mesh configuration and source–receiver geometry described in Sections 3.2 and 3.3, we simulate wave propagation in three models (Fig. 12): (a) a truncated vertical sandwich (e.g. Fohrmann et al. 2004), (b) a flower structure (e.g. Twiss & Moores 2004), and (c) a tomographic model combined with a flower structure. In the truncated sandwich, the low velocity zone is sharply terminated at 5 km depth. In the flower structure, a low velocity zone with width 2 km at the surface tapers to zero width at 9 km depth, and is smoothed with a $\sigma$ Gaussian. The tomographic model is a subregion of a derived velocity model for the San Jacinto fault zone (Allam & Ben-Zion 2012). The model was rotated to place the observed near-fault low-velocity zones in the same coordinate system of the present work. The flower structure was then added to the tomographic model with point-to-point averaging, further reducing the near-fault velocity. The final model contains a low-velocity flower structure, a 1-D gradient with depth, an overall contrast in velocity with a generally slower left side, and realistic internal 3-D heterogeneities. For all three models, vertical component velocity seismograms are shown low-passed at 6 Hz, and trapped wave sensitivity kernels are computed for the station closest to the source inside the low velocity zone. All of the seismograms for the three models are scaled at the same amplitude, making direct waveform comparisons viable.

Fig. 12(a) shows the waveforms, velocity model, and trapped wave sensitivity kernel for the truncated sandwich case. The surface waves are relatively higher amplitude, and the trapped waves relatively lower amplitude than in the other fault zone models. The wavefield is highly symmetric about the source even though the low velocity zone is entirely on one side. Several interesting
features are shown in the fault-normal (Fig. 13a) and source–receiver plane (Fig. 13b) cross-sections of the trapped wave sensitivity kernel. Below the low velocity zone (gold lines), the wavefield propagates with typical body wave banana-doughnut sensitivity. Once the wavefield impinges on the bottom boundary of the low velocity zone, it becomes narrowly confined and begins to reflect on the lateral boundaries. There is some sensitivity outside of the fault zone, but it decays rapidly with distance. In the source–receiver-plane view, the kernel resembles that of a surface wave within the low velocity zone with many visible side lobes. Additionally, there is a sharp bend in the sensitivity kernel precisely at the lower fault zone boundary.
The seismic wavefield in the vicinity of fault zone structures with bimaterial interfaces and low velocity zones is perturbed in systematic ways. In contrast to direct body waves, which have little or no sensitivity to fault zone structures, sensitivity kernels for head waves are restricted to a bimaterial interface and are concentrated in the faster medium. Surface wave kernels are broadly distributed laterally, while trapped wave kernels are extremely narrow, with sensitivity focused entirely inside the low-velocity fault zone layer. Because other phases tend to avoid this layer, trapped wave sensitivity kernels represent an opportunity for unique insight into the details of coherent low velocity layers acting as fault zone waveguides. 

### 4 Discussion

The seismic wavefield in the vicinity of fault zone structures with bimaterial interfaces and low velocity zones is perturbed in systematic ways. In contrast to direct body waves, which have little or no sensitivity to fault zone structures, sensitivity kernels for head waves are restricted to a bimaterial interface and are concentrated in the faster medium. Surface wave kernels are broadly distributed laterally, while trapped wave kernels are extremely narrow, with sensitivity focused entirely inside the low-velocity fault zone layer. Because other phases tend to avoid this layer, trapped wave sensitivity kernels represent an opportunity for unique insight into the details of coherent low velocity layers acting as fault zone waveguides. P body waves that are reflected within low velocity layers can also provide information about the internal structure of fault zones (e.g. Li et al. 2007; Yang et al. 2011), but these phases can be difficult to identify. The region around and below the waveguide can be imaged with body and surface waves. Tomographic imaging using traditional body and surface waves combined with fault zone head and trapped waves can provide detailed velocity models that self-consistently merge inner fault zone components with the surrounding regional structure.

In this study we have analysed the volumetric sensitivity of fault zone head and trapped waves by constructing finite frequency kernels for several idealized and realistic models. We simulated numerically highly accurate seismograms that fit well analytical solutions at frequencies up to 12 Hz for stations inside the low velocity zone. The simulations produce the commonly observed S-trapped waves, but not P-trapped waves which are likely associated with narrower low velocity zones than those included in this study. The kernels also provide information to be translated from wiggles on seismograms into volumetric sensitivities within a given reference model (e.g. Tromp et al. 2005; Liu & Tromp 2006; Sieminski et al. 2007a,b). The kernels themselves are therefore the building blocks for a tomographic inversion, providing insights into which types of seismic phases (wiggles) should be used in order to target specific features within a reference model. While many of the results in this paper have been known previously through theoretical efforts (e.g. Ben-Zion 1989, 1998) and from numerical simulations (e.g. Igel et al. 1997, 2002), the sensitivity kernels provide an important new step toward tomographic inversions.

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**Figure 13.** Cross-sections in the fault-normal (left) and source–receiver (right) planes for the truncated sandwich (a, b), flower structure (c, d), and tomographic (e, f) models in Fig. 13. The trapped waves propagate as direct body waves before becoming confined to the low velocity zone; this is most pronounced in the truncated sandwich. The yellow lines in (a, b) denote the boundaries of the truncated sandwich.
Our main findings from the fault zone sensitivity kernels can be summarized as follows:

1. Sensitivity of $P$ head wave is primarily on the fast side of the fault but is also sensitive to portions of the slow side (Figs 4c and 9b).

2. Sensitivity of $S$ head wave is dominantly on the fast side of the fault (Fig. 4d), and the kernel is narrower than that of the $P$ head wave, as expected.

3. Sensitivity for trapped waves is confined to fault zone (Fig. 9d). This is in stark contrast to the banana-doughnut appearance of body wave kernels.

4. Surface waves can be abruptly generated from trapped waves (Fig. 9c). Surface wave measurements for earthquakes in the fault zone are comparably sensitive to shallow surface structure outside the fault zone and to low-velocity structure inside the fault zone.

5. At lower frequencies, trapped waves may be influenced by shallow structure in the fault zone in a complex manner. For example, the red fringes near the surface in Fig. 10(b) show that an increase in the Vs velocity in this region will result in a delayed traveltime of the trapped wave. Fault-normal cross-sections of kernels (Fig. 10c) indicate that trapped waves at lower frequencies are increasingly sensitive to structure outside the fault zone, especially on the fast side of the fault.

The kernels for various seismic phases produced by fault zone structure have complementary sensitivity. Head wave kernels have sensitivity primarily in the faster medium in an area near the fault interface. Sensitivity kernels for $P$ and $S$ waves arriving immediately after head waves indicate that these phases are sensitive to the slower medium. While direct body waves recorded at stations far away from the fault show typical banana-doughnut behaviour and lack sensitivity to fault zone structure, trapped wave kernels are sensitive solely to the low velocity fault zone layer. Within this layer, trapped waves exhibit high sensitivity directly along the source-receiver path surrounded by zones of alternating sensitivity corresponding to the Fresnel zones (Fig. 10b). The width of these zones decreases as a function of frequency, as does the fault normal width of the kernel, though their number increases. Trapped waves have more complicated patterns of sensitivity in more complex models. When the source is located outside the low velocity zone, trapped waves will propagate through the surrounding medium with sensitivity as a body wave before becoming focused inside the fault zone (Figs 13a and b). Although boundary smoothing has little effect on the sensitivity of trapped waves, sharply bounded fault zones will more efficiently generate surface waves in the surrounding media (and head waves on the slower side).

We envision utilizing sensitivity kernels to improve models of fault zones in two different approaches. The first approach would be to parameterize the reference model at a similar scale to the mesh used in the simulation. Then the kernels, weighted with waveform measurements, show exactly how to perturb the reference model in order to improve the fits between observed and synthetic waveforms. This approach has been effective at regional scales (e.g. Chen et al. 2007; Fichtner et al. 2009; Tape et al. 2009). The use of fault zone waves creates additional challenges due to the targeted frequency range of 1–10 Hz and the inclusion of sharp interfaces in the reference model.

A second approach would be to maintain the highly simplified representation of the fault zone as a three-part model: a slow fault zone in between two faster blocks. Just as in the previous approach, the kernels would remain similar to the model, but the model update would be restricted to only a few parameters such as fault zone thickness and the velocities of each medium. This would allow for the inclusion of the full complexity of the waveforms and sensitivity kernels; however, the update to the reference model would discard this complexity and instead reflect a bias toward simpler models. This scheme can help resolving the trade-offs among model parameters (e.g. Ben-Zion et al. 2003). Once the best simplified model is derived from the available data, it can be merged with the regional structure with a few additional inversions using the first approach. The two schemes presented above could be alternated iteratively within a single inversion framework.

Any work incorporating fault zone sensitivity kernels to constrain tomographic inversions must establish at the outset which model parameters are fixed and which will be allowed to vary. The kernels presented in this work are for perturbations in volumetric elastic structure, yet it is clear that sharp interfaces play a major role in the generation of head, trapped, and other waves. Furthermore, real fault interfaces are complex surfaces with non-planarity expressed at a variety of scales, although they tend to be approximately planar in structures of large fault zones (e.g. Ben-Zion & Sammis 2003, and references therein). Ideally one could include the topography of these interfaces as unknown parameters in the inversion (e.g. Dahlen 2005; Liu & Tromp 2006, 2008). However, allowing both the boundary topography and the volumetric seismic velocities to vary increases non-uniqueness; characterizing appropriate solutions is more challenging with a larger number of free parameters.

Incorporating fault zone head and trapped waves into a full-waveform tomographic inversion utilizing also body and surface waves remains a tantalizing target for future work, with potential for unprecedented resolution. Crucial to such work involves (1) densely instrumented fault zones (e.g. Ben-Zion et al. 2015), (2) reasonable 3-D reference models (e.g. Allam & Ben-Zion 2012) and (3) understanding the sensitivity patterns of the various phases. The work done in this paper on the third ingredient is an important step towards full waveform tomography of fault zone regions with maximal sensitivity to different structural elements sampled by body, surface and fault zone waves.

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